CONSIDERATION OF ROUGHNESS OF A SURFACE OVER WHICH FLOW OCCURS

E. A. Shkvar

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Many pieces of technical equipment involve turbulent flows over a rough surface. More stringent requirements for calculating the characteristics of modern devices has raised ever greater interest in such flows, creating a need for their study and creation of an applicable semiempirical theory.

It is well known [1-3] that the effect of roughness of the surface over which flow occurs on the characteristics of a turbulent flow can be considered in the expression for turbulent viscosity ε over the thickness of the shear layer y by some non-negative function of y of the form $\varepsilon = \varepsilon(y + \Delta y)$. From similarity and dimensionality theory [4, 5] as well as experimental results [5-7] it follows that the effect of roughness can be characterized by the dimensionless complex $h^+ = hv_*/\nu$, where h is the equivalent height of the elements of a dense-packed sand surface, in which roughness of an arbitrary geometry and structure occur [5]; $v_* = \sqrt{\tau_w/\rho}$ is the dynamic velocity; τ_w is the tangent stress on the surface; ρ is the liquid density; $\nu = \mu/\rho$ is the kinematic molecular viscosity coefficient; μ is the dynamic viscosity.

The establishment of the functional relationship between the parameters $\Delta y^+ = \Delta y v_*/\nu$ and h^+ is of practical importance, since h^+ contains the linear dimension h, which characterizes the geometry of the roughness. Various approaches to determination of this relationship are known. Thus, in [1, 2] the function $\Delta y^+ = \Delta y^+(h^+)$ was defined empirically. A more reliable approach was used in [3, 8], the essence of which consists of defining the function $\Delta y^+(h^+)$ by comparison of known empirical relationships for the velocity profile with similar functions obtained theoretically with certain assumptions regarding the distributions of turbulent viscosity and shear stress over the thickness of the flow. However generalization of the logarithmic velocity distribution law in the region near the wall [3] is valid only for well developed roughness [1, 7], and with transition to a hydrodynamic smoothness regime it leads to a false definition of the turbulent viscosity due to the nonzero value of $\lim\limits_{h^+\to 1} \times \Delta y^+ = -2.468$. Thus, use of the approach of [3] for solution of the combined problem, in which transitions from a regime with manifestation of roughness to a hydrodynamically smooth flow and back demands refinement of the expression for $\Delta y^+(h^+)$ in the roughness regime transition region and satisfaction of the asymptotic condition $\Delta y^+(h^+ \to 1) \to 0$ at $h^+ < h_1^+$, where h_1^+ is the upper limit of the hydrodynamic smoothness regime.

The attempt made to circumvent these difficulties in a similar approach in [8] led to various expressions for description of the turbulent viscosity and various methods for determining the roughness function depending on the manifestation regime. The comparison of calculations with expressions from [1-3] shown in Fig. 1 (curves 1-3, respectively) indicates the significant divergence in the predictions of the form of the function $\Delta y^+ = \Delta y^+(h^+)$ by the various authors.

1. The present study will develop the approach used in [9] to construct a continuous algebraic model for turbulent flows over a rough surface. The calculation of turbulent viscosity on the basis of this model has significant advantages over other models [2, 3, 8, 10-13], since it permits derivation of approximate analytic expressions to describe the velocity distribution in the wall area and the external regions of the flow, as well as construction of a numerical method for calculation of the boundary layer on the basis of the continuous representation of turbulent viscosity and a unified approach to consideration of the effect of surface roughness in all its manifestations. The approximate analytical solutions thus determined permit refinement of the function $\Delta y^+(h^+)$ appearing in ε, together with effective specification of the initial velocity profile in the numerical method for calculating turbulent flows. Meanwhile the numerical calculation makes it possible to test the reliabiity of the premises used in the semiempirical theory and the refinements made. Assuming invariance

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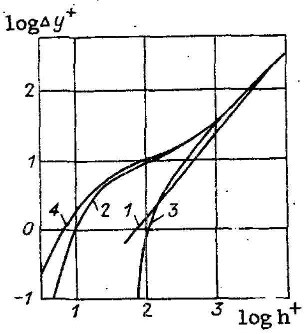

Fig. 1

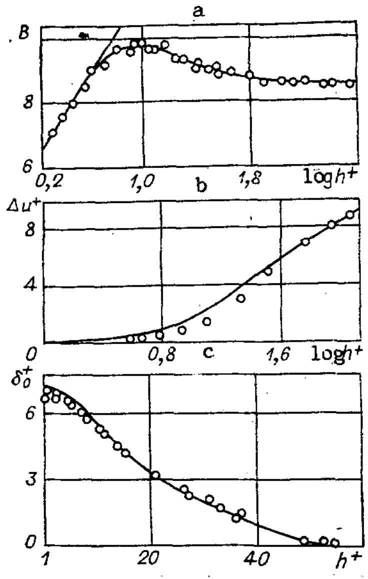

Fig. 2

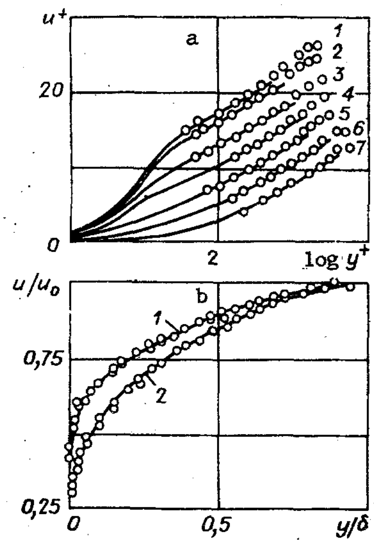

Fig. 3

of the roughness function relative to the pressure gradient, heat-mass exchange, etc., as has been confirmed experimentally and by the calculations of [2, 3, 13, 14], the function $\Delta y^+(h^+)$ was obtained for gradient-free flows.

The turbulent viscosity as calculated using a semi-empirical expression from [9], which is identical over the thickness of the boundary layer and has been employed successfully for calculation of gradient [9] and gradient-free flows [15] on a smooth surface,

$$\varepsilon = \rho\chi\Delta v_*\gamma(\eta)\,\mathrm{th}\,\frac{l\sqrt{\tau_*}}{\chi\Delta}, \tag{1.1}$$

where ε is the turbulent viscosity coefficient, $\chi = 0.0215$ is an empirical constant, ℓ is the length of the mixing path, $\eta = y/\delta$ is a dimensionless coordinate, δ is the thickness of the shear layer, $\gamma(\eta) = \sqrt{1-\eta}$ is a function which considers the effect of intermittancy of the flow, $u^+ = u/v_*$ is the averaged longitudinal velocity component, dedimensionalized to the scale of the "wall law," $u_0^+ = u(\delta)/v_*$; $\Delta = \int\limits_0^\delta (u_0^+ - u^+)\,dy$ is the Clauser length parameter, τ_* is the dimensionless shear strength in the vicinity of the wall: $\tau_* = 1 + \Phi\eta$ for $\Phi \geqslant 0$, $\tau_* = 1/(1-\Phi\eta)$ for $\Phi < 0$; $\Phi = (\delta/\tau_w)\frac{dp}{dx}$; $\frac{dp}{dx}$ is the longitudinal pressure gradient. The effect of roughness is considered in the semi-empirical expression for the mixing path length by the function $\Delta y^+(h^+)$:

$$l = k(y + \Delta y)\,\mathrm{th}\,\frac{\mathrm{sh}^2[\chi_1(y^+ + \Delta y^+)]\,\mathrm{th}(\mathrm{sh}^2[\chi_2(y^+ + \Delta y^+)])}{k(y^+ + \Delta y^+)\sqrt{\tau_*}}, \tag{1.2}$$

where $k = 0.4$, $\chi_1 = 0.072$, $\chi_2 = 0.223$ are empirical coefficients.

2. To obtain relationships describing the velocity profile, the Boussinesq expression $\frac{\partial u}{\partial y} = \frac{\tau(\Phi, \eta, \Delta_1)}{\mu + \varepsilon}$ was integrated with use of Eqs. (1.1), (1.2) for the various regions of the turbulent flow in accordance with the approach developed in [9, 15]. To describe the velocity distribution in the transition region the expression

$$u_*^+ = \frac{1}{\chi_1}\left[(1 + p^+y^+)\,\mathrm{th}[\chi_1(y^+ + \Delta y^+)] - \mathrm{th}(\chi_1\Delta y^+)\right] + \frac{p^+}{\chi_1^2}\ln\frac{\mathrm{ch}[\chi_1(y^+ + \Delta y^+)]}{\mathrm{ch}(\chi_1\Delta y^+)}, \tag{2.1}$$

was found, where $p^+ = (\nu/\rho v_*^3)\frac{dp}{dx}$ is the longitudinal pressure gradient dedimensionalized to the scale of the "wall law"; u_*^+ is the value of the velocity u^+ calculated with Eq. (2.1), which is valid for any pressure gradient, can be generalized as an approximation to the viscous sublayer, and is applicable in the range $0 \leqslant y^+ \leqslant y^*$; y^* (the coordinate at which the transition and logarithmic regions join) can be determined from the condition of smoothness of the velocity profile at the juncture point

$$y^* = \frac{\operatorname{ch}^2[\chi_1(y^* + \Delta y^+)]}{k\sqrt{1 + p^+ y^*}} - \Delta y^+,$$

which simplifies significantly for gradient-free flows:

$$y^* = \begin{cases} \frac{1}{k}\operatorname{ch}^2[\chi_1(y^* + \Delta y^+)] - \Delta y^+ = y_1^* - \Delta y^+ & \text{at} \quad \Delta y^+ < y_1^*, \\ 0 & \text{at} \quad \Delta y^+ \geqslant y_1^*. \end{cases}$$

Here $y_1^* \approx 26$ is the value of y^* for flow on a smooth surface, obtained by substitution of $\Delta y^+ = 0$ in the expression for y^*. To calculate the velocity profile in the logarithmic distribution law region the expression

$$u_1^+ = \frac{1}{k}\ln(y^+ + \Delta y^+) + C_1 \quad \text{for} \quad p^+ = 0; \tag{2.2}$$

$$u_2^+ = \frac{2}{k\sqrt{-1 - p^+\Delta y^+}}\operatorname{arctg}\sqrt{\frac{1 - p^+ y^+}{-1 - p^+\Delta y^+}} + C_2 \quad \text{for} \quad p^+\Delta y^+ < -1; \tag{2.3}$$

$$u_3^+ = \frac{1}{k}\ln\frac{\sqrt{1 - p^+ y^+} - \sqrt{1 + p^+\Delta y^+}}{\sqrt{1 - p^+ y^+} + \sqrt{1 + p^+\Delta y^+}} + C_3 \quad \text{for} \quad 0 > p^+\Delta y^+ \geqslant -1; \tag{2.4}$$

$$u_4^+ = \frac{\sqrt{1 - p^+\Delta y^+}}{k}\ln\frac{\sqrt{1 + p^+ y^+} - \sqrt{1 - p^+\Delta y^+}}{\sqrt{1 + p^+ y^+} + \sqrt{1 - p^+\Delta y^+}} + \frac{2}{k}\sqrt{1 + p^+ y^+} + C_4 \tag{2.5}$$

$$\text{for} \quad 0 < p^+\Delta y^+ \leqslant 1;$$

$$u_5^+ = \left[\sqrt{1 + p^+ y^+} - \sqrt{p^+\Delta y^+ - 1}\operatorname{arctg}\sqrt{\frac{p^+ y^+ + 1}{p^+\Delta y^+ - 1}}\right] + C_5 \tag{2.6}$$

$$\text{for} \quad p^+\Delta y^+ > 1,$$

were obtained, wherein C_k (the constant of the logarithmic law) is defined from the conditin of continuity of the velocity profile at the point of juncture of the transition and logarithmic regions

$$u_k^+(y^*) = u_*^+(y^*).$$

Equations (2.2)-(2.6) are valid for the range $y^* \leqslant y^+ \leqslant y^{**}(y^{**} = \eta_1\delta^+,\ \delta^+ = \delta v_*/\nu)$. To find η_1 (the dimensionless coordinate of the point of merger of the logarithmic and external regions) we use the condition of smoothness of the velocity profile at the merger point

$$\eta_1 = \frac{\chi\Delta_1}{k}\sqrt{\frac{1 - \eta_1}{\tau_*}}$$

($\Delta_1 = \Delta/\delta$ is the dimensionless Clauser parameter). To calculate the velocity distribution in the external region relationships obtained in [9] for flow on a smooth surface can be used, since the turbulent viscosity in this region is not a function of roughness, Eqs. (1.1), (1.2).

Comparison of Eq. (2.2) with Nikuradze's experimental relationship [4], containing the parameter h^+,

$$u_1^+ = \frac{1}{k}\ln\frac{y^+}{h^+} + B(h^+)$$

at the point y^{**} allows specification of the function $\Delta y^+(h^+)$

$$\Delta y^+ = \frac{1}{\chi_1}\operatorname{arcth}(\chi_1\Delta u^+) \quad \text{at} \quad h^+ \leqslant h^*; \tag{2.7}$$

$$\Delta y^+ = h^+\exp(-kB(h^+)) \quad \text{at} \quad h^+ > h^*,\ h^* = y_1^*\exp(k[C_1 + 2.98]); \tag{2.8}$$

$$\Delta u^+ = \frac{1}{k}\ln h^+ - B(h^+) + C_1, \tag{2.9}$$

where Δu^+ is the shear function of the logarithmic portion of the velocity profile due to the effect of roughness.

Equations (2.7)-(2.9) can be used to calculate gradient-free and gradient flows with any roughness manifestation regime if the function $B = B(H^+)$ is known. For the present study, in place of the piecewise representation of this function used in [2, 3], we will use an approximation of Nikuradze's experimental function which is identical for all three roughness manifestation regimes

$$B = C_1 + s\,\mathrm{th}\,\frac{\ln h^+}{ks},\; s = \frac{2.98}{1 - 87\left(\frac{\ln h^+}{8}\right)^{2.03}\left(1 - \frac{\ln h^+}{8}\right)^{8.386}}. \tag{2.10}$$

Figure 2 shows calculations (lines) and experimental data (points) of Nikuradze [4, 5] of the functions: $B = B(h^+)$, Eq. (2.10) (a); $\Delta u^+ = \Delta u^+(h^+)$ Eq. (2.9), (b). Since there are no experimental results for the function $\Delta y^+ = \Delta y^+(h^+)$, to establish the correspondence of calculations with Eqs. (2.7), (2.8) to the experiments of [14] (Fig. 2c), the dependence of viscous sublayer thickness on roughness was used, dedimensionalized to the scale of the "wall law" and related to Δy^+ by the expression

$$\delta_0^+(h^+) = \delta_1^+ - \Delta y^+(h^+),$$

where δ_1^+ is the thickness of the viscous sublayer for a flow above a smooth surface. The function $\Delta y^+(h^+)$, calculated directly from Eqs. (2.7), (2.8) is shown in Fig. 1 (line 4). The good agreement between velocity distributions in flows over rough surfaces constructed with Eqs. (2.1)-(2.10) (lines) with experimental profiles (points) is shown in Fig. 3: a) experiments of [16], 1-7 correspond to $\Delta y^+ = 0$, 1.2, 4.65, 7.63, 11.83, 18.04, 45.42; b) experiments of [6] (1, 2, $h^+ = 81.6$ and 1191).

3. Equations (1.1), (1.2), (2.7)-(2.10) were used in a numerical calculation of turbulent flows. The calculation method is based on a system of integral relationships for conservation of mass and momentum with Boussinesq closing

$$\frac{d\delta^*}{dx} = \frac{v_0}{u_0} + (\delta - \delta^*)\frac{1}{u_0}\frac{du_0}{dx}; \tag{3.1}$$

$$\frac{d\delta^{**}}{dx} = \frac{c_f}{2} - (2\delta^{**} + \delta^*)\frac{1}{u_0}\frac{du_0}{dx}; \tag{3.2}$$

$$\frac{\partial u}{\partial y} = \frac{\tau(\Phi, \eta, \Delta_1)}{\mu + \varepsilon}, \tag{3.3}$$

where δ^* is the displacement thickness, δ^{**} is the momentum loss thickness, $c_f = 2\tau_w/\rho u_0^2$ is the local friction coefficient, v_0 (transverse component of the average velocity on the external border of the boundary layer) is determined by integration of the equation

$$\frac{\partial v}{\partial y} = \frac{1}{u_0}\left(v\frac{\partial u}{\partial y} - v_*^2\frac{\partial \tau_+}{\partial y} - u_0\frac{du_0}{dx}\right); \tag{3.4}$$

where x is the longitudinal coordinate, τ_+ is the dimensionless shear stress, the distribution of which over shear layer thickness is specified by an exponential function [15]. To calculate ε, Eq. (1.1) was used. The thickness of the boundary layer is defined by the expression $\delta = u_0^+\delta^*/\Delta_1$. Calculation of flow characteristics is then a matter of numerical solution of a Cauchy initial-boundary problem. For the initial conditions we use known (from experiment, for example) values of δ_0^*, δ_0^{**}, c_{f0}, together with velocity of the fundamental flow $u_0 = u_0(x)$ and the liquid viscosity ν. Boundary conditions are specified in the form

$$u = v = 0 \text{ for } y = 0,\; u = u_0 \text{ for } y = \delta.$$

In the first calculation step the values of δ_0^*, δ_0^{**}, c_{f0} are used to construct the initial velocity profile by numerical integration of Eq. (3.3) using a fourth order Runge–Kutta method, as a result of which an equivalent roughness height h is chosen for the given flow. In the second and following calculation steps sequential steps from a previous to a subsequent section are made using difference analogs of Eqs. (3.1), (3.2), permitting determination of δ_i^* and δ_i^{**}. Using the known values of the integral thicknesses at each calculation section, a velocity is constructed in order to determine the parameters c_{fi}, δ_i, Δ_{1i}, by performing iterations over the two variables c_{fi}^j, Δ_{1i}^j, until the conditions

$$\left|\frac{c_{fi}^{j+1} - c_{fi}^j}{c_{fi}^j}\right| < Q,\; \left|\frac{\Delta_{1i}^{j+1} - \Delta_{1i}^j}{\Delta_{1i}^j}\right| < Q,$$

are simultaneously satisfied, where $Q = 1\cdot 10^{-3}$ is the calculation accuracy criterion, i is the number of the section, and j is the number of the iteration. Calculation of each section is completed by integration of Eq. (3.4) to determine the value of v_0 required for the step to the next section. Calculations were performed for flows with various regimes of roughness manifestation. The calculations (lines) are compared with experiment for values of c_f, δ^*,

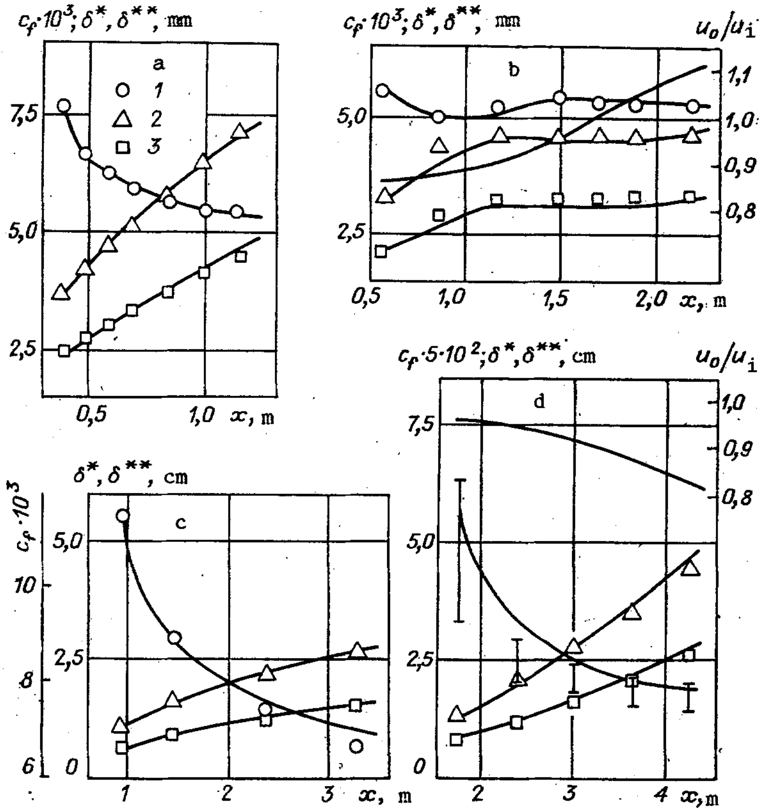

Fig. 4

and δ^{**} (points 1-3) in Fig. 4. Shown are comparisons with experimental data of Betterman [2] [u_0 = 30 m/sec, $\nu = 1.44 \cdot 10^{-5}$ m²/sec, dp/dx < 0 (a)]; Coleman [u_i = 26.4 m/sec, $\nu = 1.51 \cdot 10^{-5}$, dp/dx < 0 (b)]; Liu et al. [u_0 = 0.15 m/sec, $\nu = 0.96 \cdot 10^{-6}$, dp/dx = 0 (c)], Scottorn and Power [u_i = 30.5 m/sec, $\nu = 1.49 \cdot 10^{-5}$ m²/sec, dp/dx > 0 (d)]. Here u_i is the velocity of the incident flow. Figure 4a, b, d illustrates well developed manifestation of roughness, while Fig. 4c shows the transitional roughness regime. By selecting the velocity profile in the initial section for given experiments the following values were found for the equivalent roughness height: $h = 1.38 \cdot 10^{-3}$, $7.4 \cdot 10^{-4}$, $1.5 \cdot 10^{-2}$, $9.69 \cdot 10^{-3}$ m (Fig. 4a-d). The completely satisfactory correspondence of the calculated turbulent flow characteristics with the experimental results allows the conclusion that the expressions obtained herein are sufficiently reliable. The approach described above for consideration of the effect of roughness is free of the shortcomings of [3, 8] and is more universal than that of [2], since the calculation expressions were obtained directly from the semi-empirical turbulence model with minimum use of empirical information. The model and calculation method developed can be used for calculation of more complex hydrodynamic flows.

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EFFECT OF A DISPERSED COMPONENT ON THE NATURE OF HEAT EXCHANGE DURING FLOW OF A HETEROGENEOUS JET AROUND A BARRIER

G. F. Gorshkov

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Flow in a turbulent nonisothermal heterogeneous jet is characterized by considerable velocity [1, 2] and temperature disequilibrium [3] ($u_s \neq u$ and $T_s \neq T$, where u_s, T_s and u, T are velocity and temperature of dispersed and gas components). As was shown in [4], an impurity is not passive, and it leads to suppression of jet turbulence (a result of interphase exchange by pulse and heat). Nonetheless, during reaction of a heterogeneous jet with a barrier orientated along the normal to the running flow, a significant increase is observed in heat emission characteristics in the vicinity of the point of deceleration [5] (for a single-phase jet an increase in heat exchange is typical with an increase in the intensity of turbulence [6]). The intensity of the change in heat emission in this case is a result of velocity and temperature disequilibrium for flow in jets, and it depends on a number of factors (temperature, concentration, phase condition of the dispersed impurity, etc.) and on the nature of the reaction of the dispersed component with the barrier surface [7]. There are numerous experimental data devoted to this. Apart from work in [5, 7], attention is drawn to [8] where an increase is also noted in the heat flow (by a factor of 1.4) at the deceleration point for a plane cylindrical end and a hemisphere. The aim of the present work is a study of the effect of a dispersed component on heat exchange with jet flow around a barrier.

1. Experimental Procedure and Processing of Data. The study was carried out in an experimental unit consisting of an electrothermal source, a coordinated barrier device, a protective flap ahead of it with a mechanism for retension in the upper position, and also an automatic system for switching on the apparatus and switching off the source of supply.

High temperature jets were created by means of an electric arc heater with a partially fixed arc length. Use was made of a nozzle of the following geometry: outlet section diameter $d_a = 9\cdot10^{-3}$ m, critical cross-section diameter $d_* = (5.5\text{-}6)\cdot10^{-3}$ m. Introduction of a dispersed impurity into the jet was accomplished in the discharge space directly beyond the critical cross section. In order to feed particles use was made of a powder doser of the vibration type.

Convective heat flow during operation of the jet on a plane boundless barrier located perpendicular to the running flow, was determined by the exponential method [9] for measuring

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